

SINGULAR PERTURBATION CONTROL FOR COORDINATION OF CONVERTERS IN A FUEL CELL SYSTEM

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Abstract - In this paper, the converters coordination problem of a fuel cell system involving a hydrogen fuel cell (FC) with supercapacitors (SCs) for applications with high instantaneous dynamic power is considered. A singular perturbation approach is used to solve the control problem under unknown variable load, where the convergence of the voltage controller is analyzed by using Lyapunov theory. Simulation results show the performances of the proposed approach against a significant FC-SCs benchmark.

Keyword - Fuel cell, supercapacitors, power management, energy-shaping, singular perturbation approach.

1 PROBLEM STATEMENT

In this paper we consider the problem of the power management of an hydrogen Fuel Cell (FC) system associated to a reversible impulse energy source (the supercapacitors) due to the development increasing of electric and hybrid vehicles since 2009. The FC must deliver a slowly varying current, not more than 4A/s for a 0.5kW/12.5V FC [3], and 10A/s for a 20kW/48V FC [4] as example. That is why the FC needs to be associated with other sources (here the supercapacitors) which supply short pulse energy and fill the temporary failure of the FC [1, 2, 5]. Solving the problem leads to a design challenge consisting to choose power components, to design an adapted architecture and to define the appropriate associated control strategy [8].

The parallel architecture is the most suitable structure: the components have lower stress, the management of power is easy and is more reliable [9, 10]. The parallel structure with two converters (Fig. 1) (see [12–14]), which associates a static converter with each source, allows perfect control of the power flow of each source. However, its drawbacks are the inevitable losses associated with each static converter and a higher cost. This paper is dedicated to the study of the structure with two converters shown in Fig. 1.

Three major objectives of the problem statement are the respect of FC dynamics (mainly limited by the time response of the air compressor), control of the storage device (SCs) state of charge and the power

response (positive or negative) required by the load. It means that the current delivered by the FC must have smooth behavior in order to ensure its live time, while the SCs provide the load power transient. The hybridization of the electrical supply is highly recommended to limit the dynamic effects and a special attention is also required for the shut-down and start-up procedures to preserve the fuel cell performances and enhance its durability. Therefore, it seems clear that the DC bus regulation is managed by the supercapacitors.

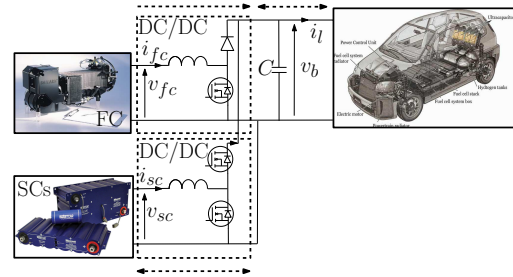


Fig. 1. Two converters parallel structure studied in this work.

To reach the objectives, high-performance controllers are readily available based on the system state [9, 17], fuzzy logic [18, 19], proportional-integral controllers [3, 11], RST controller [20], passivity [21], flatness [22, 23] or model predictive control [15].

Alternative approach exist such as optimal control

[7], dynamic programming [24] or empirical control associated with a multi objective genetic algorithm optimization [6] that has been applied for the supervisory power train control problem in charge sustaining hybrid electric vehicles. However, these approach are based on the *a priori* knowledge of the power load, thus real-time control is not straightforward implementable.

The frequency decoupling method using two cascaded loops of currents and voltages proposed in [11] allows to have :

The SCs supply power transient, while the FC supplies the mean power (This slow FC dynamics increases the system life time).

Modification of the power load induces a change in the DC bus voltage.

The gains are tuned to ensure the closed-loop system stability, although it has not been theoretically proved.

In the work of [16], a passivity based approach is adopted where the closed-loop system stability is proved. However, to overcome the difficulty of the known load, the authors have proposed to use an observer estimating the unknown resistance load or an integral action.

In this paper, a controller based on singular perturbation approach ([28, 33]) is proposed to respect the slower FC dynamics, control of the storage device (SCs) state of charge and the power response required by the load with a DC bus regulation. This solution permits to avoid the use of a PI controller loop which is not very robust to load variation [11] and the use of an observer or an integral action [16]. Moreover this approach is well adapted to the problem of FC-SCs control where the FC and SCs currents must be slow and fast respectively. A stability proof of the system is given.

The remaining of the paper is organized as follows : In section II, the idea of the singular perturbation (SP) approach is applied to the Nexa Ballard fuel cell system to design the proposed controller with a stability proof. Section III reports the simulation results.

2 SINGULAR PERTURBATION APPROACH

The control scheme problem is to drive the load voltage under the main constraint of slow variation of fuel cell current i_{fc} and the secondary constraint of super capacitor voltage V_{sc} regulation. In order to do these a singular perturbation approach [28, 33] is used, so firstly a brief recall on such approach is recall and after that the control design of fuel cell and super capacitor system is present in this section.

2.1 BRIEF RECALLS ON SINGULAR PERTURBATION

Let us consider the following nonlinear system:

$$\dot{x} = f(x, z, \varepsilon) \quad (1)$$

$$\varepsilon \dot{z} = g(x, z, \varepsilon) \quad (2)$$

with $x \in R^m$, $z \in R^n$, ε a small positive parameter and f, g two analytical vector fields of appropriate dimension. Roughly speaking, x can be seen as the slow state and z as the fast variable. Nevertheless, this must be quickly clarified in the next and some assumptions and theoretical developments must be recalled. First of all, it is usual to decompose the system (1)-(2) in decoupled two time scales dynamic. For this, it is important to be able to compute the so-called slow manifold $z = \phi(x, \varepsilon)$, this manifold is the z behavior when the fast transient time is finished ("outside the boundary-layer"). The slow manifold ϕ must verify the following equation:

$$\varepsilon \dot{\phi}(x, \varepsilon) = g(x, \phi(x, \varepsilon), \varepsilon) \quad (3)$$

where $\phi(x, \varepsilon) = \sum_{i=0}^{\infty} \alpha_i(x) \frac{\varepsilon^i}{i!}$ is computed iteratively [30]. For example the so-called frozen solution verify:

$$0 = g(x, \alpha_0(x), 0)$$

Condition is requested for the α_0 existence:

Assumption A.1: The Jacobian $\left\{ \frac{\partial g(x, z, 0)}{\partial z} \right\}$ is regular in the considered state space $x \in D_x \in R^M$ and $z \in D_z \in R^n$.

This assumption is directly linked to the implicit function theorem and in nonlinear case more than one solution is possible and in this case a fold appears, this is outside the scope of this recall. Now, it is important to know if the system (1)-(2) converges on a slow manifold this is given by the well known Tikhonov's theorem [29] but before recall the theorem it is necessary to analyze the fast dynamic on the boundary-layer. For this, a new state variable $\eta = z - \phi$ is introduced and η converges rapidly to zero if the system behavior converges on the slow manifold.

The η dynamics is equal to

$$\dot{\eta} = \frac{1}{\varepsilon} g(x, \phi(x, \varepsilon) + \eta, \varepsilon) - \frac{\partial(\phi(x, \varepsilon), \varepsilon)}{\partial t} \quad (4)$$

Setting $\varsigma = \frac{t}{\varepsilon}$, (4) may be rewritten

$$\frac{\partial \eta}{\partial \varsigma} = g(x, \phi(x, \varepsilon) + \eta, \varepsilon) - \varepsilon \frac{\partial \phi(x, \varepsilon), \varepsilon}{\partial \varsigma} \quad (5)$$

Assumption A.2 The system (5) is locally in η and uniformly in x exponentially stable.

Hereafter, the Tikhonov's Theorem without considerations of time domain and existence and uniqueness of the solution (for example Lipschitz conditions are implicit).

Theorem 2.1 *Under the assumptions A.1 and A.2 and for $\varepsilon \in R^+$ sufficiently small, after a transient t_1 the dynamics (1)-(2) evolves on a slow dynamic and his dynamic is equal to:*

$$\dot{x} = g(x, \phi(x), \varepsilon) \quad (6)$$

In many applications, (6) is approximated at first order (the frozen solution of ϕ)

$$\dot{x} = g(x, \alpha_0(x), 0) \quad (7)$$

As in the considered application the dynamics are too dynamically closed, some fast actuators (high-gain feedback) are used [31].

Considering, for the seck of simplicity, the following dynamical system

$$\dot{\chi} = \tilde{f}(\chi, \zeta) \quad (8)$$

$$\dot{\zeta} = \tilde{g}(\chi, \zeta) + \tilde{\beta}(\chi)u \quad (9)$$

with $\chi \in R^m$, $\zeta \in R^n$, $u \in R^n$ and $\tilde{\beta}$ regular for all χ . Then setting, for example $u = -\frac{1}{\varepsilon}\tilde{\beta}(\chi)^{-1}(\zeta - \alpha_0(\chi))$ the dynamic become:

$$\dot{\chi} = f(\chi, \zeta) \quad (10)$$

$$\varepsilon \dot{\zeta} = \varepsilon \tilde{g}(\chi, \zeta) - (\zeta - \alpha_0(\chi)) \quad (11)$$

Dynamics (10)-(11) are similar to the dynamics (1)-(2), thus, it is possible to use the theorem 2.1 and the slow dynamic of (10)-(11) in first approximation is equal to

$$\dot{\chi} = f(\chi, \alpha_0(\chi)) \quad (12)$$

Remark 2.1 *In this paper, we often restrict our purpose to the first approximation of the slow manifold i.e. $\phi \simeq \alpha_0(x)$, nevertheless, for example, when the system behavior is too closed to a fold it is necessary to do a better ϕ 's approximation.*

2.2 FUEL CELL-SUPERCAPACITORS MODELS

2.2.1 Power electronic's modeling

In this paper, the modeling of the various elements is the same as in numerous studies [20, 21].

Fuel cell modeling The model used is a static model [25–27] where the FC voltage v_{fc} is computed according to the current stack i_{fc} by a 5th order polynomial function of the stack current i_{fc} as shown in Fig. 2, in order to improve the simulation time and used a simple equation without loss of accuracy.

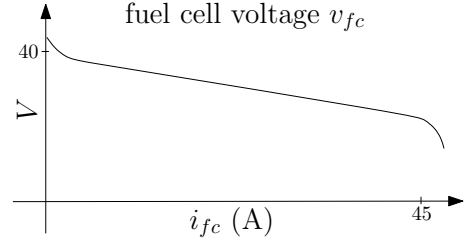


Fig. 2. Fuel cell voltage vs current.

FC boost converter To use the FC system in an electric power system, the FC must be connected the FC to a fixed DC bus voltage. In this case, the FC voltage must be increased, because it is often less than the DC bus voltage. The boost converter represented in Fig. (3) is controlled by binary input $w_1(t)$.

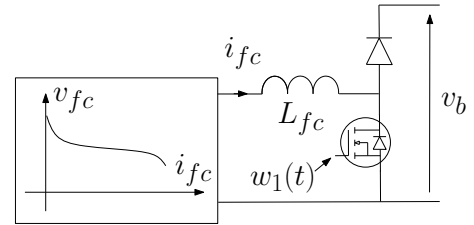


Fig. 3. Boost converter.

Defining α_1 as the duty cycle of control variable $w_1(t)$, this subsystem can be represented by its average model (here, the switches are regarded as ideal):

$$\begin{aligned} \frac{di_{fc}}{dt} &= \frac{1}{L_{fc}}(- (1 - \alpha_1) v_b + v_{fc}) \\ \frac{dv_b}{dt} &= \frac{1}{C}((1 - \alpha_1) i_{fc} - i_l) \end{aligned} \quad (13)$$

where v_b is the DC link voltage, v_{fc} is the FC voltage, i_l is the DC current delivered to the load and i_{fc} is the FC current.

The product between control input α_1 and state variable v_b shows a non-linear behaviour of the converter.

SCs boost converter SCs can be charged or discharged; therefore the storage elements are connected to the DC bus through a reversible power converter.

The SCs used here have a constant capacity (C_{sc}) and negligible losses. They are associated with an inductance (L_{sc}) and a boost converter as shown in Fig. (4). Two types of operations are possible : a buck operating mode when SCs receive energy from the DC bus, and a boost operating mode when SCs supply energy to the DC bus.

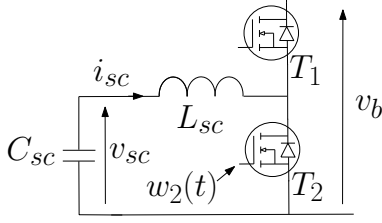


Fig. 4. Boost converter.

We define α_2 as the duty cycle of control variable $w_2(t)$. The second sub-system is represented by an average model as follows:

$$\begin{aligned} \frac{di_{sc}}{dt} &= \frac{1}{L_{sc}} \left(-(1 - \alpha_2) v_b + v_{sc} \right) \\ \frac{dv_{sc}}{dt} &= -\frac{i_{sc}}{C_{sc}} \end{aligned} \quad (14)$$

DC bus and load model Fig. (5) shows the model of the DC bus and the load. In our work, the load is modeled by a $(R_l L_l)$ circuit, whose resistance value R_l varies according to the power required by the load. E_l represents the electromotive force of an electrical machine. When the electric load operates as a generator, this electromotive force (e.m.f., E_l) is positive. The average model is:

$$\begin{aligned} \frac{dv_b}{dt} &= \frac{1}{C} \left((1 - \alpha_1) i_{fc} + (1 - \alpha_2) i_{sc} - i_l \right) \\ \frac{di_l}{dt} &= \frac{1}{L_l} \left(-R_l i_l + v_b \right) \end{aligned} \quad (15)$$

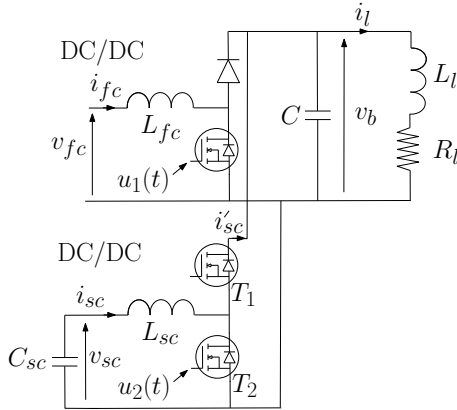


Fig. 5. DC bus and load model.

2.2.2 Two time scales complete model

Complete model It follows that the complete “fuel cell -supercapacitors” system is represented by the 5th order non-linear state space model :

$$\begin{aligned} \dot{x}_1 &= \frac{(1-\alpha_1)x_4 + (1-\alpha_2)x_5 - x_3}{C} \\ \dot{x}_2 &= -\frac{x_5}{C_{sc}} \\ \dot{x}_3 &= \frac{-R_l x_3 + x_1}{L_l} \\ \dot{x}_4 &= \frac{-(1-\alpha_1)x_1 + x_2}{L_{fc}} \\ \dot{x}_5 &= \frac{-(1-\alpha_2)x_1 + x_2}{L_{sc}} \end{aligned} \quad (16)$$

with state space $x(t) = [x_1; x_2; x_3; x_4; x_5]^t = [v_b; v_{sc}; i_l; i_{fc}; i_{sc}]^t$, control inputs $u(t) = [u_1; u_2]^t = [1 - \alpha_1; 1 - \alpha_2]^t$, measures $y(t) = x$ and $z(t) = v_{fc}$, and electromotive force E_l as a perturbation (d). At this model, it is applied, similarly to [31] and as it is recalled in the previous section two fast actuators. This fast actuators are two PI controllers which assign respectively I_{fc} and I_{sc} to a desired value I_{fc}^* and I_{sc}^* . Here, the dynamics of both PI are chosen such that assumptions A.1-A.2 are verified and outside the boundary-layer the dynamic is on the slow manifold and the system behavior is given by equation (7).

2.2.3 Slow reduced model

In order to present the slow reduced dynamics (16) in input-state representation I_{fc}^* is considered as the input u_1 and I_{sc}^* as the input u_2 and from (7) the reduced slow system is given by:

$$\begin{aligned} \dot{x}_1 &= \frac{1}{C} \left(\frac{z_3}{x_1} u_1 + \frac{x_2}{x_1} u_2 - x_3 \right) \\ \dot{x}_2 &= -\frac{u_2}{C_{sc}} \\ \dot{x}_3 &= \frac{-R_l x_3 + x_1}{L_l} \end{aligned} \quad (17)$$

with $x = [x_1; x_2; x_3]^t = [v_b; v_{sc}; i_l]^t$; control inputs $u = [u_1; u_2]^t = [i_{fc}; i_{sc}]^t$, measures $y = [v_b; v_{sc}; i_l]^t$ and $z = [i_{fc}; i_{sc}; v_{fc}]^t$. It is important to note that the model (17) is valid only if all its closed loop dynamic are slower than the dynamic of the fast actuator, here the PI controller which assign the current I_{fc} and I_{sc} , this will be a constrain in the design of the slow controllers.

2.3 SLOW CONTROL DESIGN

The desired equilibrium point are the following one $x^* = [v_b^*; v_{sc}^*; \frac{v_b^*}{R_l}]^t$, with v_b^* and v_{sc}^* the DC bus and SCs desired voltages. As the constraint on I_{fc} is such that its dynamic must be very slow (i.e. $\frac{dI_{fc}}{dt} < 4As^{-1}$) u_1 is considered as a slow input and has only a slow effect on the convergence of the dynamic (17). Consequently the gain on the u_1 loop must be very small and the variation of the component of u_1 must be filtered directly or implicitly. From these considerations u_1 is designed as follow

$$u_1 = I_{lm} - \frac{C_{sc}}{T} e_{sc} \quad (18)$$

Where I_{lm} is an average of the current in the load (in reality a load's current filtered with a low pass filter). Moreover, $e_{sc} = V_{sc} - V_{sc}^*$ and T are chosen in order that the feedback on e_{sc} will be also sufficiently slow. Here it is important to note that e_{sc} is not filtered because V_{sc}^* is a constant and V_{sc} is proportional to the I_{sc} integral and then implicitly filtered. Now, as u_1 is a slow input the second input

u_2 can be faster than u_1 but slower than the current loops driven by both PI fast actuators, so u_2 is:

$$u_2 = \frac{x_1}{x_2} \left[\frac{C}{\tau} e_b - \frac{z_3}{x_1} (I_{lm} - \frac{C_{cs}}{T} e_{sc}) + x_3 \right] \quad (19)$$

with $e_b = V_b - V_b^*$ and τ is chosen in order than the feedback on e_b is slower than both PI fast actuators but faster than the loop on e_{sc} . From (18) and (19), it is possible to set the following proposition.

Proposition 2.1 *The system (17) under the controls (18) and (19) is locally exponentially stable for V_b^* and V_{sc}^* constants with a bias on $x_2 = V_{sc}$.*

Proof 2.1 *Choosing the following Lyapunov function $V_T = V_1 + V_2 + V_3$ with $V_1 = 0.5(x_3 - i_{lm})^2$, $V_2 = 0.5e_b^2$ and $V_3 = 0.5e_{sc}^2$, the differentiation of V_2 gives:*

$$\dot{V}_2 = -\frac{e_b^2}{\tau}$$

Consequently e_b converges exponentially to zero uniformly with respect to e_{sc} and e_{sc} . The differentiation of V_1 is equal to:

$$\dot{V}_1 = (x_3 - i_{lm}) \frac{-R_l x_3 + x_1}{L_l}$$

Setting $e_i = x_3 - i_{lm}$ and the fact that $i_{lm} = V_b^*/R_l$ one has

$$\dot{V}_1 = e_i \frac{-R_l e_i + e_b}{L_l}$$

as e_b converge to zero independently of e_i then e_i also converges exponentially to zero and this uniformly with respect to e_{sc} . Finally V_3 gives:

$$\dot{V}_3 = -e_{sc} \left(\frac{\frac{x_1}{x_2} \left[\frac{C}{\tau} e_b - \frac{z_3}{x_1} (I_{lm} - \frac{C_{cs}}{T} e_{sc}) + x_3 \right]}{C_{cs}} \right)$$

Since e_b converges to zero and x_3 to i_{lm} , then after such convergence one has

$$\dot{V}_3 = -\frac{e_{sc}}{C_{sc}} \left(\frac{z_3}{x_2} \frac{C_{sc}}{T} e_{sc} - \frac{z_3}{x_2} I_{lm} + \frac{x_1}{x_2} I_{lm} \right)$$

Which is locally exponentially stable for $x_2 > 0$, with a bias convergence of e_{sc} to $\frac{(z_3 - x_1) I_{lm} T}{C_{sc} z_3}$. Moreover T must be chosen in order that the dynamic of e_{sc} is slower with respect to PI fast actuators.

Remark 2.2 *It is important to note that the bias may be canceled by adding an adaptive skim. This will be done in a future work. Moreover due to practical implementation, the control is digital and some problems appear ([32], [34]).*

3 SIMULATION RESULTS

To test and validate the performances of the proposed FC controller (18-19), a dedicated FC Control Benchmark is proposed by Fig. (6) where the reference DC bus voltage is set equal to 48V and the load current varies between 4.8 and 15A. This power cycle is representative of a vehicle power demand, where the load requirement consists in raising and lowering power edges between 240 and 720W.

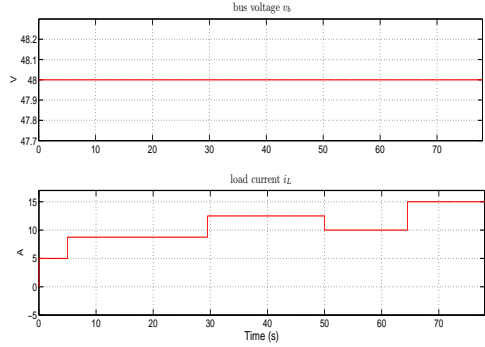


Fig. 6. Fuel Cell Control Benchmark.

Concerning the load, the proposed FC controller uses only the measurement of load current while the load is an unknown variable.

The block diagram scheme of the proposed controller adopted in this work and used in simulation is given by Fig. 7. It is composed of three subsystems : one fast actuator for the FC based on a PI controller, second actuator to manage the current of the SCs based on a IP controller, and the proposed singular perturbation approach to control the DC bus voltage and state of charge of the SCs.

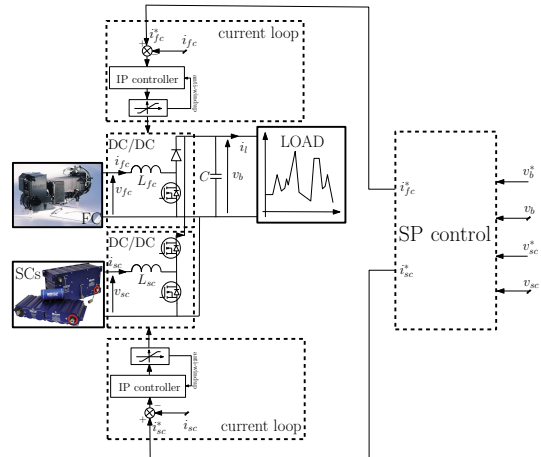


Fig. 7. Singular Perturbation structure.

Fig. (8) shows the response of the FC hybrid system according to the reference trajectories of the Benchmark (Fig. 6). It can be remarked that a very good tracking of the DC bus (Fig. 8.a) is ensured by the proposed fast controller (19) with a smooth response of the FC current¹ (Fig. 8.e) during fast power demands of the load (Fig. 8.c) so as to maintain the FC state of health. Moreover, the energy balance is well achieved, characterized by a SC voltage (Fig. 8.b)) value reaching the desired set point (24V) at steady state by the proposed slow controller (18).

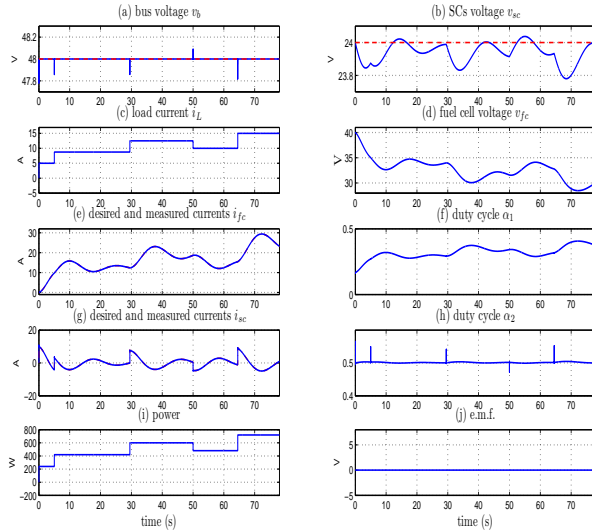


Fig. 8. Simulation result with an unknown load.

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¹The FC rate of change is nearly equal to 4A/s (or 10A/s, as for an FC example given by [3, 4])

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